Using Excel and GSP as computer-based mathematical tools to assist secondary students setting up models with exponential and cosine functions.

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Abstract

Although Vietnamese secondary students are quite good in solving problems with given mathematical models or partial modelling, but they feel difficult in creating their own mathematical models to solve non-routine realistic problems. Influenced by a focus on modelling with computer-based mathematical tools (CBMT), we will try to describe and give some situations that involved Vietnamese students in setting up and using models to solve real-world problems. We introduced the cycle of modelling with exponential and sinusoidal functions by using Microsoft Excel (2007) and GSP (version 5) as computer-based mathematical tools. In this article, we use Microsoft Excel as an automatically CBMT to set up exponential model, and the GSP as dynamically CBMT to make sinusoidal model. The results showed that setting up models with CBMT created an active learning environment in the mathematics classrooms and secondary students to construct a specific cycle with some concrete steps to solve real-life problems.

1. Introduction

Secondary Vietnamese students highly appreciate their learning mathematics in schools. They try their best to learn mathematics under the pressure of family and society. Most of the studentswho participated in the PISA 2012, 2015 surveys (Program for International Student Assessment) agreed the following statement "*it is worth making an effort in learning mathematics in schools, because equipped mathematical concepts and skills will help us to perform well in our desired professions later on in life*" ([6]). Vietnamese students are proficient in core mathematical concepts and skills, the main reason is students need to study hard to overcome the summative assessments in an educational environment focused too much on "*learning to take examination*" ([9]).

A visible role for mathematical tools, including technology isdescribed in the PISA 2021 framework. The definition of mathematical literacy explicitly includes the use of mathematical tools (MT). These tools include a variety of *physical* and *digital* equipment, software and calculation devices. Computer-based mathematical tools (CBMT) are in common use in workplaces of the 21st century, and will be increasingly more prevalent as the century progresses both in the workplace and in society generally. The nature of day-to-day and work-related problems and the demands on individuals to be able to employ mathematical reasoning (both deductive and inductive) in situations where computational tools are present has expanded with these new opportunities – creating enhanced expectations for mathematical literacy ([7]).Computer-assisted instruction programs are considered a way to improve learning outcomes of students. However, little is known aboutthe schools that implement such programs as well as the effectiveness of similar information and communication technology programs ([10]).

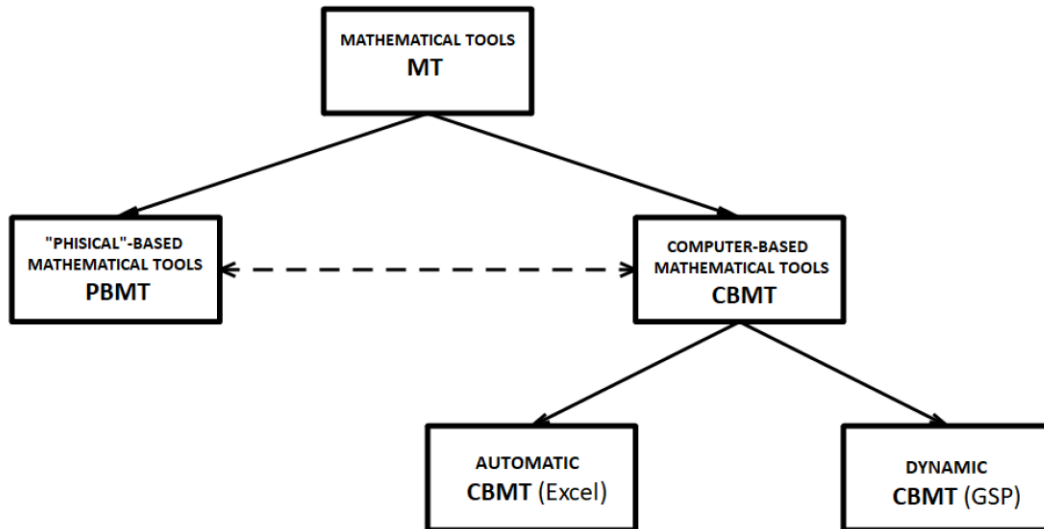The way in which we view mathematical tools is shown in Figure 1.

Figure 1. *The categories of mathematical tools including PBMT & CBMT.*

We consider an automatic CBMT as a CBMT that gives the mathematical models automatically if we input properly data and predict appropriately the models, and in a dynamic CBMT, we can drag the sliders dynamically to get the mathematical models.

In this article, we see Microsoft Office Excel (2007, [3]) as an automatic CBMT to set up an exponential model, and the Geometer's Sketchpad (GSP, Version 5, [4]) as a dynamic CBMT to set up a sinusoidal model. In the present study, a case study design was adopted as this design is appropriate for in-depth investigation of student mathematical modelling activity. This design has been argued to be suitable for investigations involving cognitive processes ([2]). There were 124 Grade 12 high school students (17-18 years old) from three classes who participated in this pilot study. They had learnt the derivative and integral of real functions, and the relationship between speed and distance of an article. They can use Microsoft Excel and dynamic software to draw the graphs of functions. The school was located in Hue City, a regional city in Vietnam ([10]).

2. Modelling with computer-based mathematical tools

In this article, a mathematical model is an exponential or function that describes a real-world problem. Modelling is the process of finding such functions with the assistance of computer-based mathematical tools. They bring the real world into the computer environment where we need to arrange the data, make and use a model. The way in which we view the process is shown in Figure 2.
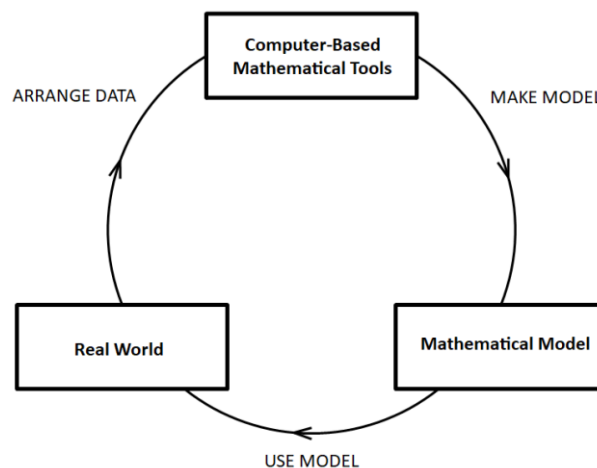
Figure 2. *The process of modelling in finding mathematical model with CBMT.*

Many of the processes studied in science involves understanding of how one quantity varies with respect to another. Finding a *function* that describes the dependence of one quantity on another is called a modelling. In the process of modelling real-life situations, in some cases of the continuous variable *x*, we can use mathematical reasoning with "physical"-based mathematical tools to set up the function *f(x)* that models the situation. But with some situations given by data, the shape of a scatter plot helps us to choose the type of curve to use in modelling the data ([8]).

To set up and use a model for real-life problems, we guide students to use a process including four specific steps as follows:

Step 1. *Arrange the table of data*. Arrange the given data into a table with two columns. First column represents the independent variable *x*. The second column is the value of the function *y = f(x)* respectively.

Step 2. *Draw the scatter plot*. Select two columns of data, use Excel (or GSP) to draw a scatter plot.

Step3. *Set up the model*. Choose the function whose graph fits the scatter plot automatically (by Excel) or dynamically (by GSP). Find out the appropriate function that models the problem.

Step4. *Use the model*. Use thefunction they have found to answer the questions posed in the problem.

3. Using Excel as an automatic CBMT to fit exponential curves to data

If a scatter plot shows that the data increase rapidly, we might want to model the data using an exponential model, that is, a function of the form

$$f(x) = Ce^{kx}$$

where *C* and *k* are constants.

3.1. Problem:Vietnam population growth

The Vietnam population data for 20th century are given in the Table 1 (Using and retrieving data from ([1]).

Use Microsoft Excel 2007 ([3]) to investigate this problem.

Table 1. *The Vietnam population data for* 1900-2000.

Year (*t*)	Population (*P*in millions)
1900	16.7
1910	18.2
1920	19.9
1930	22.3
1940	24.7
1950	26.4
1960	33.5
1970	42.6
1980	54.3
1990	68.0
2000	79.9

a) Use Excel to make a scatter plot of the data. Note that a linear model is not appropriate.
b) Find an exponential function that models Vietnam population growth.
c) Draw a graph of the function that you found together with the scatter plot in Excel. How well does the exponential model fit the scatter plot?
d) Use the model that you made to predict Vietnam population in the year 2025.

3.2. Tentative solution with the assistance of Excel

a) Arrange the data with $x = 0$ correspond to the year 1990, $x = 10$ correspond to the year 2000, then 2025 corresponds to $x = 12.5$.

Table 2. *The Vietnam population data for* 1900-2000*used for Excel*.

Year (t)	Population (P)
0	16.7
1	18.2
2	19.9
3	22,3
4	24.7
5	26.4
6	33.5
7	42.6
8	54.3
9	68.0
10	79.9

Click **Insert | Scatter**.
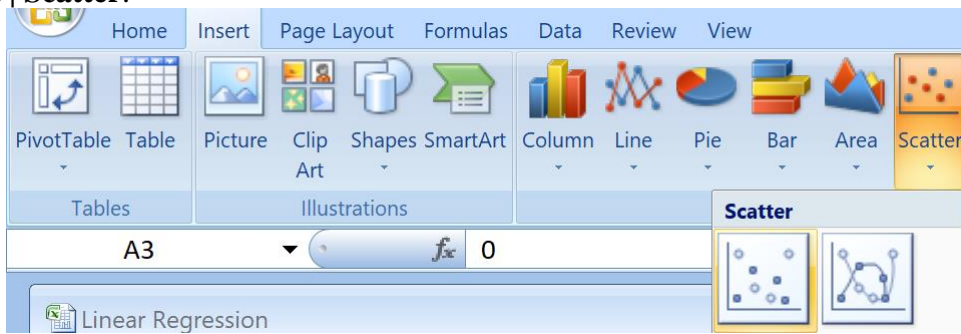
Figure 3. *Click Insert | Scatter | Scatter With Only Markers*.

A Scatter plot shows in Figure 2 made by Excel.
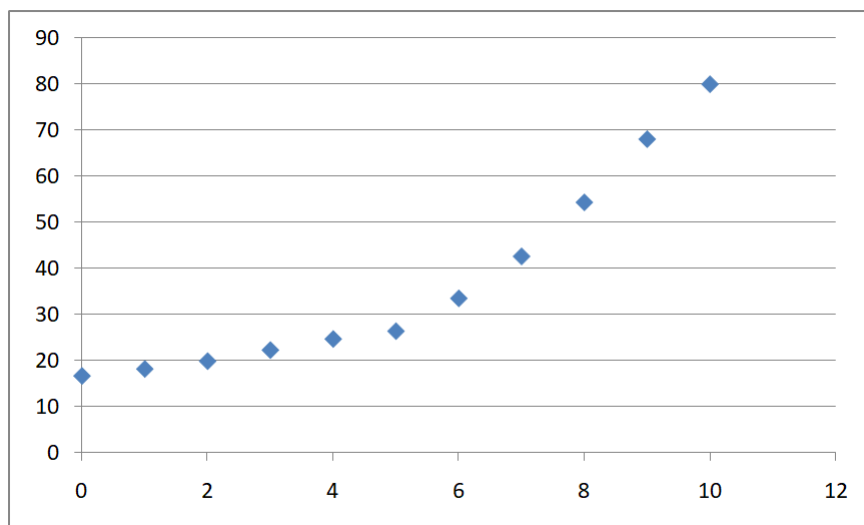
Figure 4. *Scatter plot of Vietnam population*.

b) Select the "**Chart Area**", click "**Layout | Trendline | More Trendline Options | Exponential**". To accessthe expression of the function, click "**Display Equation on chart | Close**".
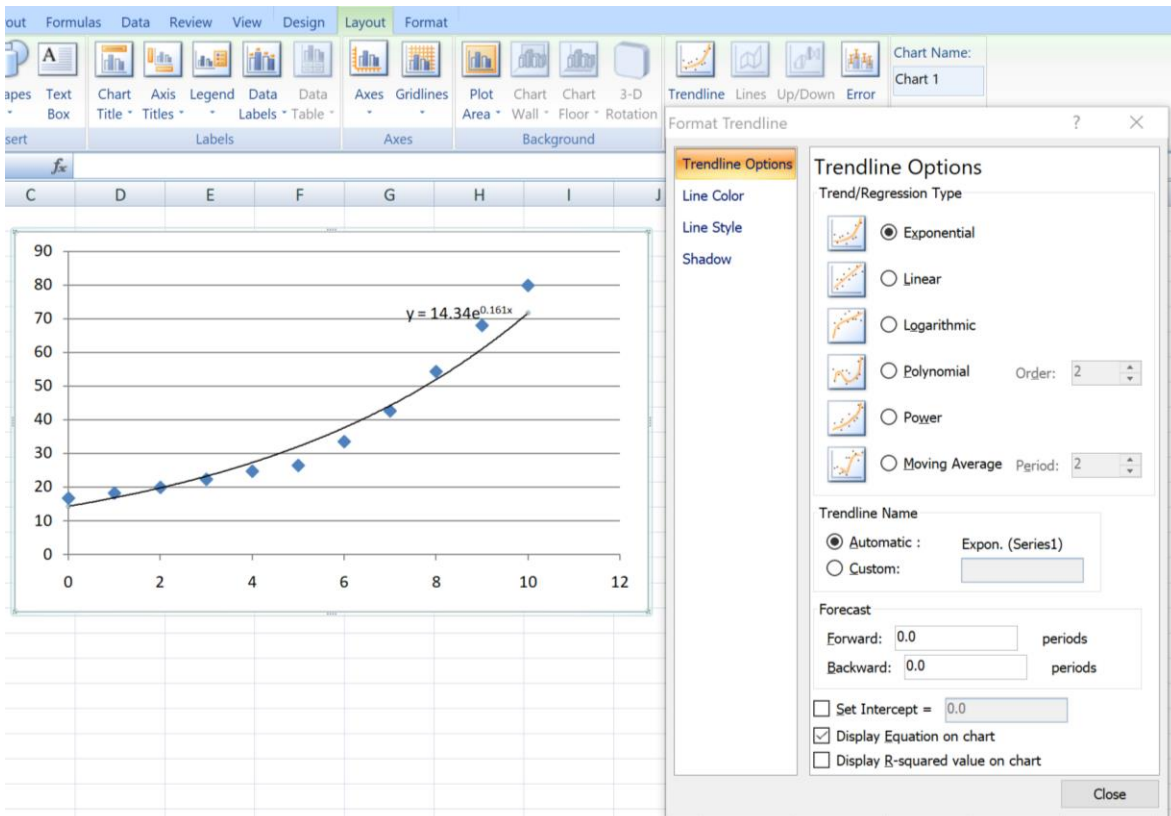
Figure 5. *Trendline Options | Exponential | Display Equation on chart | Close*.

In this case, Excel displays the expression of the exponential function automatically. We found that the function is:

$$f(x) = 14.34 \times e^{0.161x}$$

c) The graph of the exponential function together with the scatter plot have been plotted in Excel shown in Figure 6.
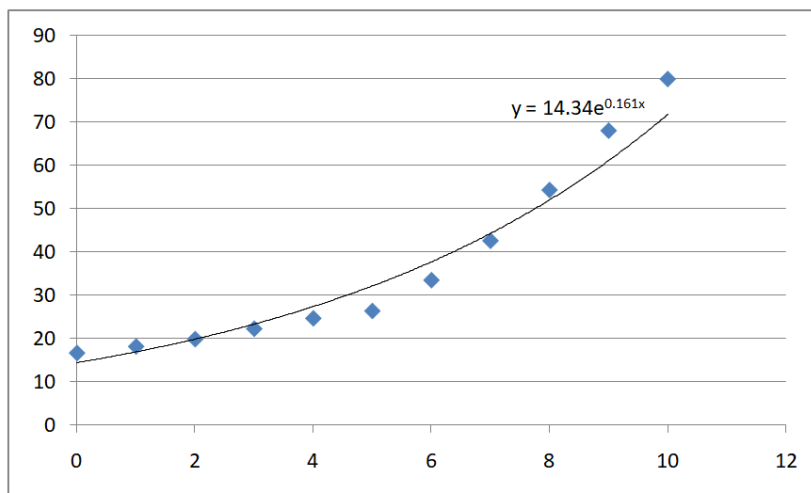

Figure 6. *The graph of the exponential function together with the scatter plot*.

From Figure 6, we see that the exponential model fit closely the scatter plot of data.

d) The exponential function that models the Vietnam population growth may be taken as:
$$f(x) = 14.34 \times e^{0.161x}$$
The year 2025 corresponds to $x = 12.5$, so we can use the found model to predict the population in year 2025:
$$14.34 \times e^{0.161 \times 12.5} = 107.29 \text{ millions}$$

3.3. Linearizing exponential data

In the problem on the Vietnam population growth, if we see that the data points (x, y) lie on an exponential curve $f(x) = Ce^{kx}$, then the points $(x, \ln y)$ will lie on a straight line. This step helps students to check their found exponential function is appropriate or not.

If $y = Ce^{kx}$

Then $\ln y = \ln C + kx$.

Let $Y = \ln y$ and $A = \ln C$; then $Y = kx + A$. This shows that $(x, Y) = (x, \ln y)$ lie on a straight line. We check the linearizing for the Vietnam population data (t, P) to obtain the data points $(t, \ln P)$ in Table 3.

Table 3. *The data points $(t, \ln P)$.*

T	P	lnP
0	16.7	2.815409
1	18.2	2.901422
2	19.9	2.99072
3	22.3	3.104587
4	24.7	3.206803
5	26.4	3.273364
6	33.5	3.511545
7	42.6	3.751854
8	54.3	3.994524
9	68	4.219508
10	79.9	4.380776

The scatter plot of the Vietnam population data $(t, \ln P)$ shows that the linearized data lie approximately on a straight line, so an exponential model should be appropriate.
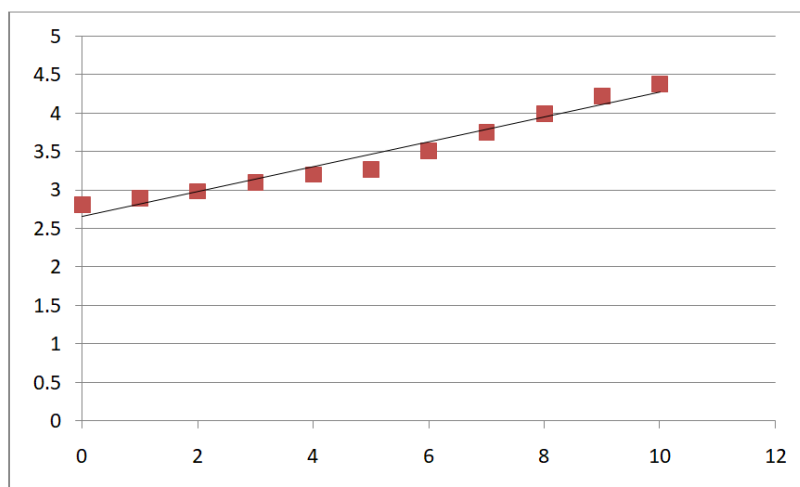

Figure 7. *The linearized data lie approximately on a straight line.*

4. Using GSP as a dynamically CBMT to fit sinusoidal curves to data

4.1. Problem: Modelling the depth of a tide
(This problem was adopted and modified from [8]).

The water depth in a narrow channel varies with the tides. Table 4 shows the water depth in meters over a 12-h period.

Table 4. *The data of the water depth over a* 12-h *period*.

Time	Depth (m)
12:00 A.M.	2.99
1:00 A.M.	3.47
2:00 A.M.	3.40
3:00 A.M.	3.41
4:00 A.M.	2.93
5:00 A.M.	2.59
6:00 A.M.	1.98
7:00 A.M.	1.74
8:00 A.M.	1.65
9:00 A.M.	1.83
10:00 A.M.	2.13
11:00 A.M.	2.62
12:00 P.M.	3.05

a) Use a computer-based mathematical tool to make a scatter plot of the data.
b) Find a function that models the water depth with respect to time.
c) If a boat needs at least 3.4 m of water depth to cross the channel, during which times can it safely do so?

4.2. Tentative solution with the assistance of The Geometer's Sketchpad (GSP)

a) First, we try to use Excel to make a scatter plot of the data.
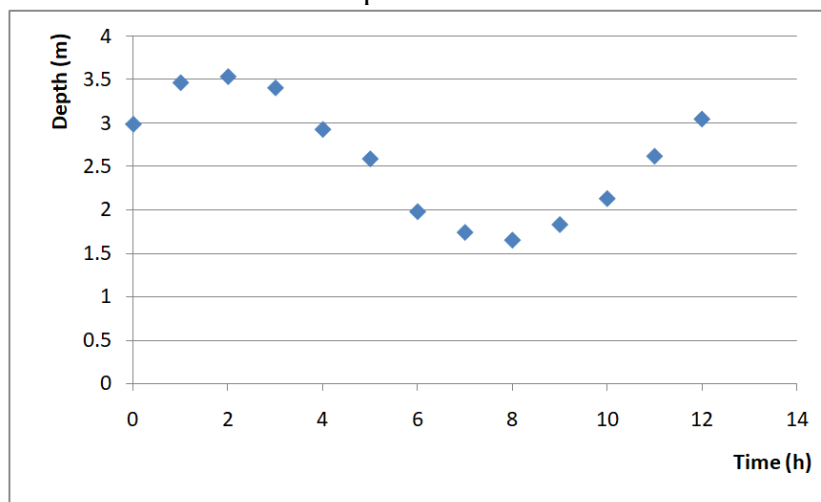

Figure 8. *Scatter plot of data in Excel.*

From the scatter plot, we see that the data appear to lie on a cosine (or sine) curve. But until now, the Excel does not have the trendline options for sinusoidal curves. So we try to use the software GSP Version 5 (The Geometer's Sketchpad, [4]) as a computer-based mathematical tool to investigate this problem.

Arrange the data into two columns of a table in GSP by click **Number | Tabulate | Add Table Data**.

Table 5. *Arranging The Table Data in GSP*.

Time	Depth
0.00	2.99
1.00	3.47
2.00	3.54
3.00	3.41
4.00	2.93
5.00	2.59
6.00	1.98
7.00	1.74
8.00	1.65
9.00	1.83
10.00	2.13
11.00	2.62
12.00	3.05

Click the **Table Data**, then **Graph | Plot Table Data**.
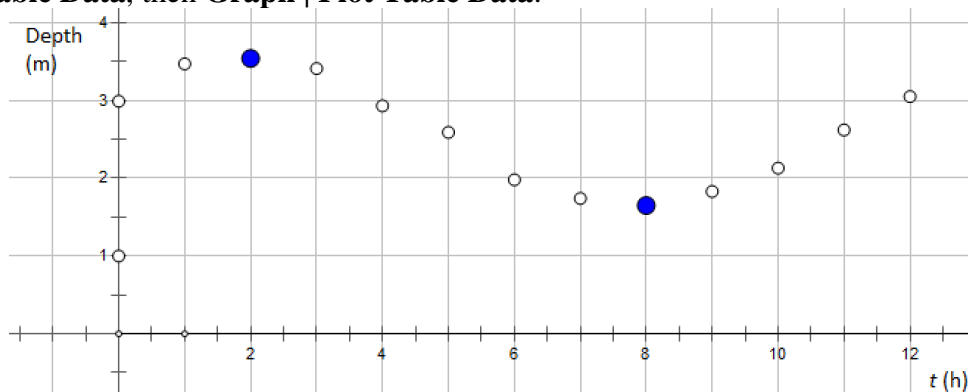
Figure 9. *Scatter plot made by GSP with two blue points are Max and Min values*.

b) To find a cosine function to model the depth of water in terms of time, first we graph $y = \cos t$, on the same graph as the scatter plot. The result in Figure 10 is not even close to the data.
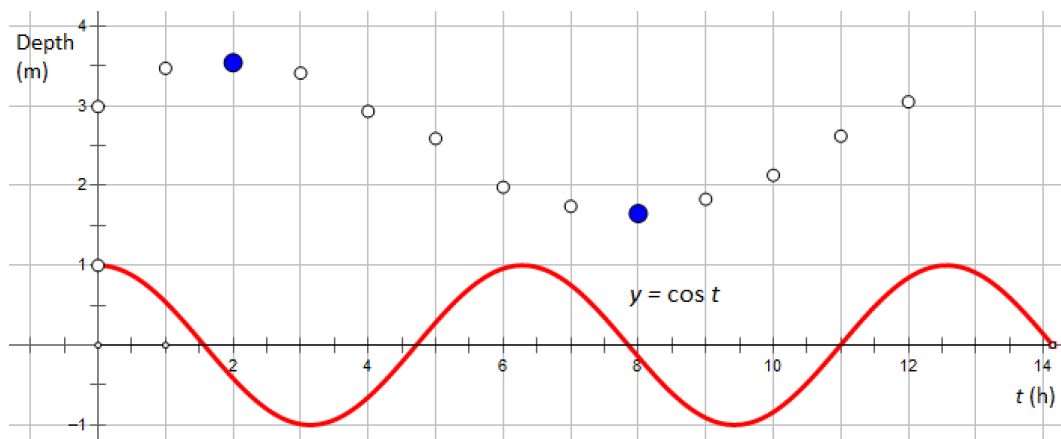

Figure 10. *The graph of* $y = \cos t$ *on the same graph of the scatter plot*.

To fit the data, we need to adjust the vertical shift, amplitude, period, and phase shift of the cosine curve. In other words, we need to find a function of the form:

$$y = a \cos\big(\omega(t - c)\big) + b$$

Calculate the vertical shift

The vertical shift b is the average of the maximum and minimum values:

$$b = \frac{1}{2}(\text{maximum value} + \text{minimum value})$$
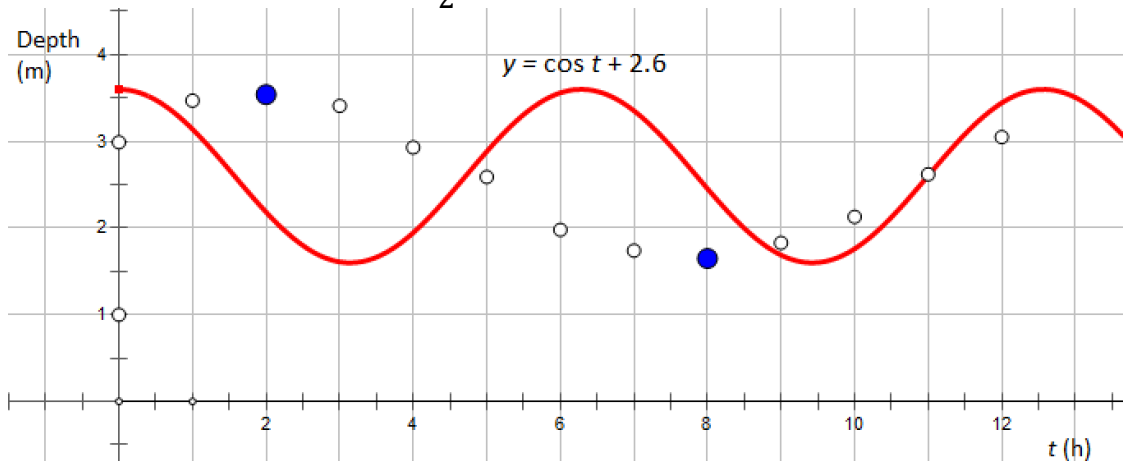
$$= \frac{1}{2}(3.54 + 1.65) = 2.6$$

Figure 11. *The graph of "$y = \cos t + 2.6$" on the same graph as the scatter plot.*

Calculate the amplitude

The amplitude a is half of the difference between the maximum and minimum values of the data:

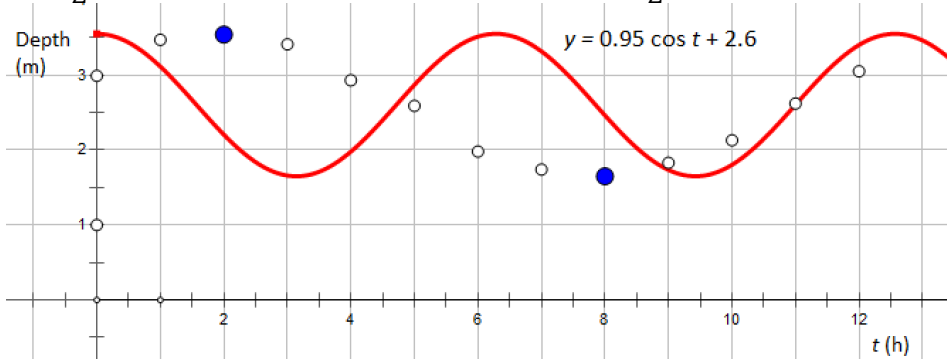$$a = \frac{1}{2}(\text{maximum value} - \text{minimum value}) = \frac{1}{2}(3.54 - 1.65) = 0.95$$

Figure 12. *The graph of "$y = 0.95 \cos t + 2.6$" on the same graph of the scatter plot.*

Calculate the period

The time between consecutive maximum and minimum values is half of one period. Thus:

$$\frac{2\pi}{\omega} = 2(\text{time of maximum value} - \text{time of minimum value})$$

$$= 2(8 - 2) = 12$$

Then we calculate the value:
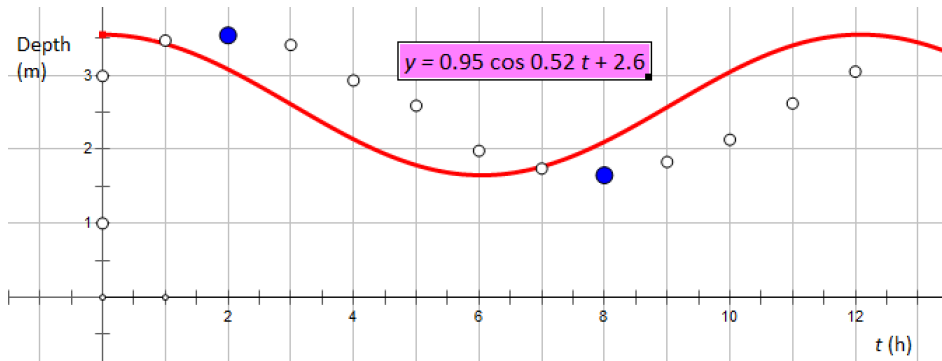
$$\omega = \frac{2\pi}{12} = 0.52$$

Figure 13. *The graph of "y = 0.95 cos 0.52 t + 2.6" on the same graph as the scatter plot.*

Calculate the horizontal shift

Since the maximum value of the data occurs at approximately $t = 2$, it represents a cosine curve shifted 2 h to the right. So, the phase shift $c = 2$.
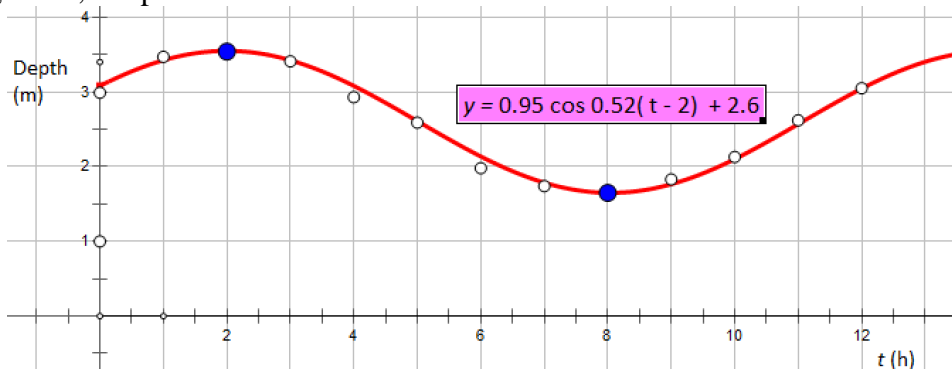

Figure 14. *The graph of function* $y = 0.95 \cos 0.52 (t - 2) + 2.6$.

The model

We have found that a function that models the tides over the given time period may be given by.

$$y = 0.95 \cos 0.52 (t - 2) + 2.6$$

A graph of the function and the scatter plot made by GSP are shown in Figure 14. We observe that the model we set up is a closelyapproximation to data.

c) We need to solve the inequality $y \geq 3.4$. Draw the horizontal line $y = 3.4$. Select two graphs and find their intersections.
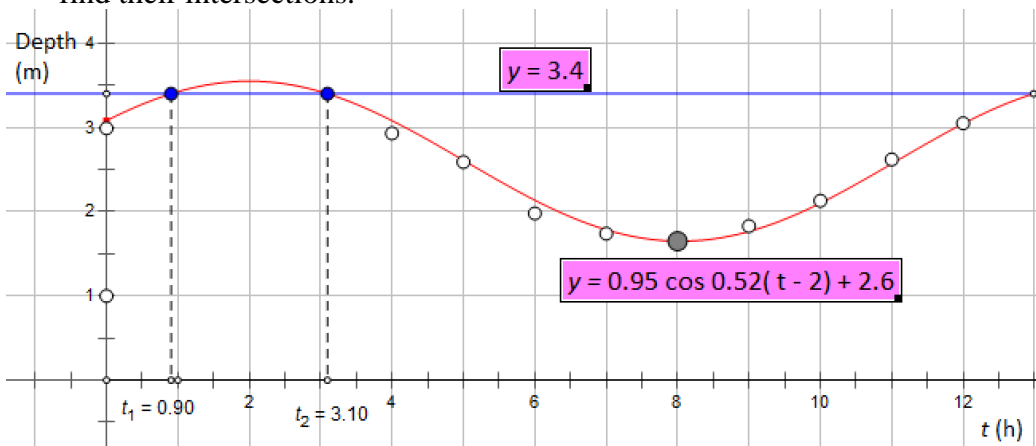

Figure 15. *Solve the inequality by graphing two functions on the same graph.*

From the graph in Figure 15, we find two intersections respectively with:
$$t_1 = 0.9 \text{ and } t_2 = 3.10$$

This corresponds to the times 12:54 A.M. to 3:06 A.M. for the boat crossing the channel safely. This is a visual solution that in our experience students find easy to seethe period of times by using GSP.

4.3. Dynamic Model
We can make four algebraic sliders for coefficients b, a, ω, c. Draw the graph of function
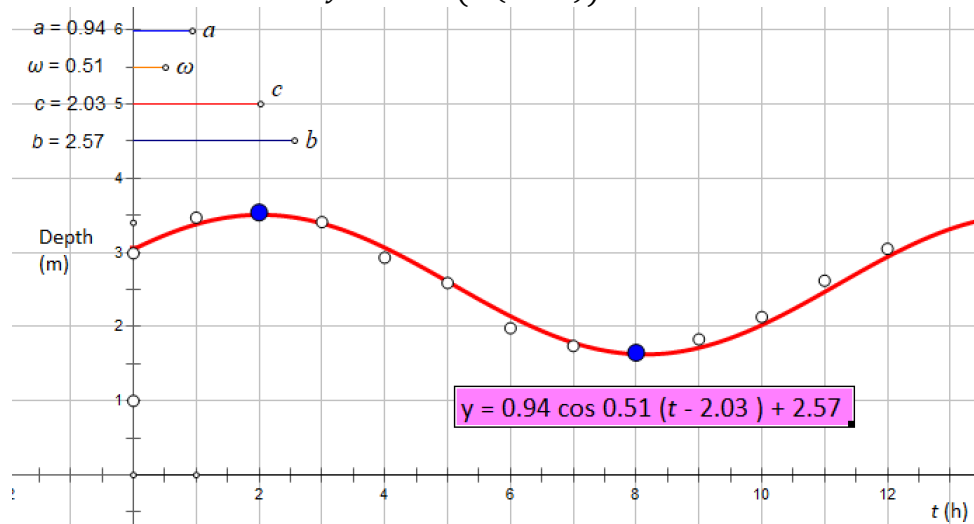$$y = a \cos(\omega(t - c)) + b$$

Figure 16. *Dynamic model for cosine functions*.

Let students drag the algebraic sliders to observe and explore the meaning and the effect of the vertical shift, the amplitude, the period and the horizontal shift to the graph of cosine functions. Adjust the four sliders to obtain the cosine function that best fits the scatter plot of data. Compare the equation of the obtained function by adjusting the sliders with the equation of the function obtained by directly calculating the coefficients.

5. Conclusion
Mathematics classrooms teachers created rich real-world situations for mathematical modelling with the assistance of computer-based mathematical tools. The implementation of the process of mathematical modelling in teaching and learning mathematics is really effective. Students can demonstrate deep understanding of a core concept by translating a real-life problem related to that concept to mathematical models.

Although Vietnamese students are quite good in solving problems with given mathematical models or partial modelling, hey feel difficult in creating their own mathematical models to solve non-routine realistic problems. That is the reason mathematics educators should focus more on the use of computer-based mathematics tools in integrating mathematical modelling and problem-solving processes with basic concepts and skills in mathematics classrooms to enhance mathematical thinking as focused in MOET mathematics curriculum ([5]). The results showed that setting up models with CBMT created an active learning environment in the classroom and students make a cycle to solve real-life problems.

6. References

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